MCV4U Limits and Derivatives: Applications

1. Mohamed is driving a motorcycle with ___________________ in the side car. Mohamed's path over \( t \) seconds is given by \( M(t) = t^3 - 2t^2 + 3t \). The side car detaches after 1 second (and since the side car has no steering of its own, its path will be along the tangent line). Will Mohamed and the side car meet up again? When?

\[
M'(t) = 3t^2 - 4t + 3
\]

\[
M'(1) = 2
\]

\[
M(1) = 2
\]

*equation of tangent:*

\[
y = 2(t - 1) + 2
\]

\[
t^3 - 2t^2 + 3t = 2(t - 1) + 2
\]

\[
t^3 - 2t^2 + 3t = 2t
\]

\[
t^3 - 2t^2 + t = 0
\]

\[
t(t^2 - 2t + 1) = 0
\]

\[
t(t - 1)^2 = 0
\]

\[
t = 0 \text{ or } 1
\]

Therefore, no, Mohamed does not meet up with the side car again after detaching at 1 second.

2. Hmedat and Clara have made cookie dough for the senior bake sale. They have made \( C(t) = \frac{1}{5}t(t + 5)(t + 3)(t + 1) \) cookies after \( t \) hours. At what rate are they making cookies after 2 hours?

\[
C(t) = \frac{1}{5}(t^2 + 5t)(t^2 + 4t + 3)
\]

\[
C'(t) = \frac{1}{5}[(2t + 5)(t^2 + 4t + 3) + (t^2 + 5t)(2t + 4)]
\]

\[
C'(2) = \frac{1}{5}[(9)(15) + (14)(8)]
\]

\[
= 49.4
\]

*Therefore, they are making cookies at a rate of 49.4 cookies per hour after 2 hours.*

3. The weekly sale of sandals \( S \) in thousands of pairs is given by \( S(t) = \frac{120t}{t^2 + 100} \), where \( t \) is the number of weeks after the introduction of this style. After how many weeks do we have a maximum value?

\[
S'(t) = \frac{(120)(t^2 + 100) - (120t)(2t)}{(t^2 + 100)^2}
\]

\[
0 = 120t^2 + 12000 - 240t^2
\]

\[
= 12000 - 120t^2
\]

\[
120t^2 = 12000
\]

\[
t^2 = 100
\]

\[
t = \pm 10
\]

*test points to find out if after 10 weeks it is a maximum value.*

*Therefore, after 10 weeks, we have a maximum sales record.*
4. Temperature affects the amount of electricity used to run the heater. The amount you run the heater affects the monthly bill. Find how much your bill will increase if the temperature is $8^\circ C$.

\[ E(T) = \frac{5T + 100}{T + 2} \] where $E$ is the energy in KiloJoules and $T$ is the temperature in $^\circ C$.

\[ B(E) = 3 + 0.2E \] where $B$ is the monthly bill.

\[
B'(T) = B'(E(T))E'(T)
\]

\[
= (0.2) \frac{(5)(T + 2) - (5T + 100)(1)}{(T + 2)^2}
\]

\[
B'(8) = 0.2 \frac{(5)(10) - (140)}{10^2}
\]

\[
= -0.18
\]

Therefore, the Bill is decreasing (not increasing) by $0.18$ at $8^\circ C$. 