MCV4U Diagnostic Review from Previous Math Courses

Show your work (you may use separate paper if you need) when you answer the following questions:

1. Determine the slope of the line passing through the following pairs of points:
   a) \((3,7)\) and \((5, -9)\)
   \[
   m = \frac{-9 - 7}{5 - 3} = \frac{-16}{2} = -8
   \]
   b) \(\left(\frac{1}{2}, 3\right)\) and \((1,4)\)
   \[
   m = \frac{4 - 3}{1 - \frac{1}{2}} = \frac{1}{1 - \frac{1}{2}} = \frac{1}{\frac{1}{2}} = 2
   \]

2. Determine the equations of the following lines:
   a) slope: 3, \(y - \text{intercept: } -5\)
   \[y = 3x - 5\]
   b) slope: 6, through \((2,18)\)
   \[y = 6(x - 2) + 18 \quad \text{or} \quad y = 6x + 6\]
   c) through \((3,5)\) and \((6, -4)\)
   \[
   m = \frac{-4 - 5}{6 - 3} = \frac{-9}{3} = -3
   \]
   \[y = -3(x - 3) + 5 \quad \text{or} \quad y = -3x + 14\]
   d) horizontal, through \((4,6)\)
   \[y = 6\]

3. Evaluate \(f(3)\)
   a) \(f(x) = -x^2 + 5\)
   \[
   f(3) = -(3)^2 + 5 = -9 + 5 = -4
   \]
   b) \(f(x) = (3x - 1)(2x + 4)\)
   \[
   f(3) = (3(3) - 1)(2(3) + 4) = (9 - 1)(6 + 4) = (8)(10) = 80
   \]

4. Expand and simplify:
   a) \((x - 5)(x + 3) + (2x - 1)(x + 2)\)
   \[
   = (x^2 - 2x - 15) + (2x^2 + 3x - 2) = 3x^2 + x - 17
   \]
   b) \((a + 5)^3\)
   \[
   = (a + 5)(a^2 + 10a + 25) = (a^3 + 10a^2 + 25a) + (5a^2 + 50a + 125) = a^3 + 15a^2 + 75a + 125
   \]
5. Factor the following expressions:
   
a) \(x^2 - 2x - 8\)
   \[= (x - 4)(x + 2)\]

b) \(2x^2 - 5x - 3\)
   \[= 2x^2 - 6x + x - 3\]
   \[= 2x(x - 3) + (x - 3)\]
   \[= (x - 3)(2x + 1)\]

c) \(x^3 - 2x^2 - 3x\)
   \[= x(x^2 - 2x - 3)\]
   \[= x(x - 3)(x + 1)\]

d) \(x^3 - 2x^2 - 5x + 6\)
   \[if \ f(x) = x^3 - 2x^2 - 5x + 6,\]
   \[f(1) = 0, so \ (x - 1) is a factor\]
   \[synthetic division gives:\]
   \[= (x - 1)(x^2 - x - 6)\]
   \[= (x - 1)(x - 3)(x + 2)\]

6. What is the domain of each of the following functions?
   
a) \(f(x) = \sqrt{x - 3}\)
   \[D(f) = \{x \in \mathbb{R} | x \geq 3\}\]

b) \(g(x) = \frac{x^2 + 3}{x - 4}\)
   \[D(g) = \{x \in \mathbb{R} | x \neq 4\}\]

c) \(h(x) = \frac{7x}{x^2 - 2x - 8}\)
   \[D(h) = \{x \in \mathbb{R} | x \neq 4, -2\}\]

d) \(j(x) = x^3 + 2x^2\)
   \[D(j) = \{x \in \mathbb{R}\}\]

7. Find the average rate of change over the interval \(0 \leq x \leq 3\) for the following functions:
   
a) \(f(x) = 3x^3 - x^2 + 5\)
   \[AROC = \frac{f(3) - f(0)}{3 - 0} = \frac{77 - 5}{3} = \frac{72}{3} = 24\]

b) \(g(x) = 2^x\)
   \[AROC = \frac{f(3) - f(0)}{3 - 0} = \frac{8 - 1}{3} = \frac{7}{3}\]
8. Estimate the instantaneous rate of change when \( x = 5 \) for the following functions:
   
a) \( f(x) = 3x^3 - x^2 + 5 \)
   
\[
IROC = \frac{f(5.01) - f(5)}{0.01}
\]

\[
= \frac{357.15 - 355}{0.01}
= 215.44
\]

b) \( g(x) = 2^x \)

\[
IROC = \frac{f(5.01) - f(5)}{0.01}
\]

\[
= \frac{32.223 - 32}{0.01}
= 22.26
\]

9. Simplify the following expressions by using exponent laws:
   
a) \( a^7 \times a^2 \)
   
\[
= a^9
\]

b) \( \frac{5b^5 \times 10b^3}{15b^4} \)

\[
= \frac{50b^8}{15b^4}
= \frac{3b^4}{b^4}
= \frac{10}{3}
\]

c) \( (-3c)^5(-2c)^2 \)

\[
= (-3)^5(c)^5(-2)^2(c)^2
= -243(4)c^7
= -972c^7
\]

d) \( (d^3x^7)^2(d^{-2}x^{-4}) \)

\[
= d^6x^{14}d^{-2}x^{-4}
= d^4x^{10}
\]

10. For \( f(x) = 4x^2 + 3 \)

   a) Determine the expression for the difference quotient \( \frac{f(a+h)-f(a)}{h} \) when \( a = 2 \)

\[
f^{IROC}(h) = \frac{4(2+h)^2 + 3 - (4(2)^2 + 3)}{h}
\]

\[
= \frac{4(2^2 + 4h + h^2) + 3 - 4(2^2) - 3}{h}
= \frac{16h + 4h^2}{h}
= 16 + 4h, \ h \neq 0
\]

   b) What can that expression be used for?
   
Finding an estimate for the IROC at \( a = 2 \).

   c) Evaluate the expression from part (a) when \( h = 0.01 \).

\[
f^{IROC}(0.01) = 16 + 0.04
= 16.04
\]

   d) What does this value represent?
   
The IROC of \( f(x) \) at \( x = 2 \) is approximately 16.04.