Sketch the graph of \( f(x) = x^3 - 9x^2 + 15x - 135 \)

**Factor:**

<table>
<thead>
<tr>
<th>METHOD 1: Factoring with Factor Theorem</th>
<th>METHOD 2: Factoring by grouping</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(9) = 0; (x - 9) \text{is a factor by factor theorem} )</td>
<td>( f(x) = x^3 - 9x^2 + 15x - 135 )</td>
</tr>
</tbody>
</table>
| \[ \begin{array}{cccc}
1 & -9 & 15 & -135 \\
9 & 9 & 0 & 135 \\
\end{array} \] | \( = x^2(x - 9) + 15(x - 9) \) |
| \( f(x) = (x - 9)(x^2 + 15) \) | \( = (x - 9)(x^2 + 15) \) |
| \( x - \text{int at 9; the rest is unfactorable.} \) | \( x - \text{int at 9; the rest is unfactorable.} \) |

\( y - \text{int: } f(0) = -135 \)

\( f(x) \text{is a polynomial, therefore, no asymptotes} \)

**First Derivative Test:**
\( f(x) \text{is differentiable and continuous:} \)
\( f'(x) = 3x^2 - 18x + 15 \)
\( 0 = 3(x^2 - 6x + 5) \)
\( = 3(x - 5)(x - 1) \)
\( \text{local max/min } x = 5, x = 1 \)

**Second Derivative Test:**
\( f''(x) = 6x - 18 \)
\( 0 = 6x - 18 \)
\( 18 = 6x \)
\( 3 = x \)

<table>
<thead>
<tr>
<th></th>
<th>( x &lt; 1 )</th>
<th>( 1 &lt; x &lt; 3 )</th>
<th>( 3 &lt; x &lt; 5 )</th>
<th>( 5 &lt; x &lt; 9 )</th>
<th>( 9 &lt; x )</th>
</tr>
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<tbody>
<tr>
<td>( f(x) )</td>
<td>-</td>
<td>-</td>
<td>-</td>
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<tr>
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|                      | negative, increasing, concave down | negative, decreasing, concave down | negative, decreasing, concave up | negative, increasing, concave up | positive, increasing, concave up |

\( x = 1 \text{ is a local max} \)
\( (1, -128) \)

\( x = 3 \text{ is an inflection point} \)
\( (3, -144) \)

\( x = 5 \text{ is a local min} \)
\( (5, -160) \)

\( x = 9 \text{ is an int} \)
\( (9, 0) \)
sketch the graph of \( f(x) = \frac{x^2 - 3x + 6}{x - 1} \).

\( f(x) \) has no \( x - \text{int} \)

(quad formula on numerator)

\( y - \text{int} \):
\( f(0) = -6 \)

no holes

\( VA: \)
\( x = 1 \)

**HA/OA: (we know it will have an OA because of the Advanced Functions rule)**

<table>
<thead>
<tr>
<th>Left:</th>
<th>Right:</th>
</tr>
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<tbody>
<tr>
<td>[ \lim_{x \to \infty} f(x) ]</td>
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<td>[ = \lim_{x \to \infty} \frac{x^2 - 3x + 6}{x - 1} ]</td>
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<tr>
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</tr>
<tr>
<td>[ = DNE(-\infty) ]</td>
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Therefore, the OA has an equation of \( y = x - 2 \) and we approach it from below. Therefore, the OA has an equation of \( y = x - 2 \) and we approach it from above.

\[
f'(x) = \frac{(2x - 3)(x - 1) - (x^2 - 3x + 6)(1)}{(x - 1)^2}
\]

\[
= \frac{(2x^2 - 5x + 3) - x^2 + 3x - 6}{(x - 1)^2}
\]

\[
= \frac{x^2 - 2x - 3}{(x - 1)^2}
\]

\[
= \frac{(x - 3)(x + 1)}{(x - 1)^2}
\]

\( f'(x) = 0 \) when \( x = 3, -1 \)

\( f'(x) \) DNE when \( x = 1 \)
\[ f''(x) = \frac{(2x - 2)(x - 1)^2 - (x^2 - 2x - 3)(2(x - 1))}{(x - 1)^4} \]
\[ = \frac{(x - 1)((2x - 2)(x - 1) - 2(x^2 - 2x - 3))}{(x - 1)^4} \]
\[ = \frac{2x^2 - 2x - 2x + 2 - 2x^2 + 4x + 6}{(x - 1)^3} \]
\[ = \frac{8}{(x - 1)^3} \]

\[ f''(x) = 0 \text{ never; undefined when } x = 1 \]

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- \( x = -1 \) is a local max \((-1, -5)\)
- \( x = 1 \) is a VA
- \( x = 3 \) is a local min \((3, 3)\)