Please be comfortable with the definitions (or at least have them written down) before each section's class (you can see the course calendar to be sure on the schedule). This will save time in class so that we can get through more examples and spend more time clarifying/understanding definitions and less time just writing them). Thank you.
Vectors as Forces (7.1)

**Def:** The Resultant Force of several forces is the single force that can be used to represent the combined effect of all the forces. The individual forces that make up the resultant are referred to as the components of the resultant.

**Def:** The Equilibrant of a number of forces is the single force that opposes the resultant of the forces acting on an object. When the equilibrant is applied to the object, this force maintains the object in a state of equilibrium (stability, balance, not changing).

**State of Equilibrium**

A set of vectors is in a state of equilibrium when the resultant is equal to the equilibrant (ie: the resultant vector is \(0^-\)).

<table>
<thead>
<tr>
<th>Three collinear vectors in equilibrium</th>
<th>Three noncollinear vectors in equilibrium</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Collinear Vectors" /></td>
<td><img src="image2" alt="Noncollinear Vectors" /></td>
</tr>
<tr>
<td>(a^- + b^- + c^- = 0^-)</td>
<td>(a^- + b^- + c^- = 0^-)</td>
</tr>
</tbody>
</table>

Another way to think of this is, for example, \(a^-\), \(b^-\), and \(c^-\) are in equilibrium if \(c^-\) is the equilibrant of \(a^-\) and \(b^-\). (or vice versa).

**Components**

Previously, we mentioned that components are certain vectors that make up another vector. (There are infinitely many components for a single vector. Consider the red vector \(v^-\) that is shown in the two diagrams below. Two options are shown in black for two possible sets of component vectors, but there are many more options).

**Think to yourself:** how many different parallelograms of black vectors could we make for a given diagonal vector \(v^-\)? (ANS: infinite)
Often, we want to look at a very specific type of components - perpendicular components; horizontal and vertical vectors.

Def: Resolution is the process of decomposing a vector into its two components. We use the notation \( \vec{f} = \vec{f}_x + \vec{f}_y \); \( |\vec{f}_x| = |\vec{f}| \cos \theta \); \( |\vec{f}_y| = |\vec{f}| \sin \theta \); \( \theta \) is the angle that \( \vec{f} \) makes with the \( x \)-axis.
The Dot Product of Two Geometric Vectors (7.3)

"Geometric" vectors refers to vectors that do not have a coordinate system associated with them.

Def: The dot product of two vectors is defined as follows:
\[ A \cdot B = |A||B| \cos \theta, \ 0 \leq \theta \leq 180^\circ \] where \( \theta \) is the angle between the two vectors.

Observations:
- the dot product is always a scalar (Why? \( |A| \) is a magnitude, \( |B| \) is a magnitude, \( \cos \theta \) is a magnitude, so the product is also a magnitude/scalar); sometimes the dot product is called the "scalar product" for this reason.
- the dot product can be negative, zero, or positive depending on the size of the angle between the two vectors
  (what angles would give a negative? ANS: angles that give \( \cos \theta < 0; 90^\circ < \theta < 270^\circ \)
  what angle would give zero? ANS: angles that give \( \cos \theta = 0; 90^\circ = \theta, 270^\circ = \theta \)
  what angles would give positive? ANS: angles that give \( \cos \theta > 0; 0 < \theta < 90^\circ, 270^\circ < \theta < 360^\circ \)
- the dot product between two perpendicular vectors is zero.

Properties of the Dot Product:

Commutative Property: \( p \cdot q = q \cdot p \)
Distributive Property: \( p \cdot (q + r) = p \cdot q + p \cdot r \)
Magnitudes Property: \( p \cdot p = |p|^2 \)
Associative Property with a scalar: \( (kp) \cdot q = p \cdot (kq) = k(p \cdot q) \)
The Dot Product of Algebraic Vectors (7.4)

Algebraic vectors are vectors expressed in component form \(a^\rightarrow = (3, 2, -5)\)

**Def:** In \(\mathbb{R}^3\), if \(a^\rightarrow = (a_1, a_2, a_3)\) and \(b^\rightarrow = (b_1, b_2, b_3)\), then \(a^\rightarrow \cdot b^\rightarrow = b^\rightarrow \cdot a^\rightarrow = a_1 b_1 + a_2 b_2 + a_3 b_3\).

(we will prove this in class)

**Observations:**
This definition ALSO shows that the dot product is a scalar. We have used \(\mathbb{R}^3\) to prove this, but it is actually true for vectors of any number of components (as long as \(a^\rightarrow\) and \(b^\rightarrow\) have the same number of components).

ie: \(a^\rightarrow \cdot b^\rightarrow = a_1 b_1 + a_2 b_2\)

for \(a^\rightarrow = (a_1, a_2), b^\rightarrow = (b_1, b_2)\)
The Cross Product of Two Vectors (7.6)

\( \mathbf{a} \times \mathbf{b} \) is sometimes called the vector product.

Unlike the dot product (scalar product), which can be used in any dimension, the cross product can only be used in \( \mathbb{R}^3 \) since the cross product in a physical representation is the vector that is perpendicular to both \( \mathbf{a} \) and \( \mathbf{b} \).

We did a question like this using the dot product a few days ago - it required a linear system that we solved, though we ended up with a parameter (we used \( t \), sometimes the book uses \( k \)). The cross product is that solution where we choose \( t = 1 \).

Note that sometimes it is hard to tell whether we are calculating \( \mathbf{a} \times \mathbf{b} \) or \( \mathbf{b} \times \mathbf{a} \); by developing a formula, we will have that taken care of.

The reason this has to do with rotation is:
for \( \mathbf{a} \times \mathbf{b} \), we are finding the vector around which \( \mathbf{a} \) would have to rotated counter-clockwise (looking down the cross product vector) in order to become collinear with \( \mathbf{b} \) using a right handed system.

Properties of the Cross Product:

- Let \( \mathbf{p}, \mathbf{q}, \) and \( \mathbf{r} \) be three vectors in \( \mathbb{R}^3 \) and let \( k \in \mathbb{R} \)
- Vector product is not commutative: \( \mathbf{p} \times \mathbf{q} = -(\mathbf{q} \times \mathbf{p}) \)
- Distributive law for vector product: \( \mathbf{p} \times (\mathbf{q} + \mathbf{r}) = \mathbf{p} \times \mathbf{q} + \mathbf{p} \times \mathbf{r} \)
- Scalar law for vector product: \( k(\mathbf{p} \times \mathbf{q}) = (kp) \times \mathbf{q} = \mathbf{p} \times (k\mathbf{q}) \)
Applications of the Dot Product and Cross Product (7.7)

Work:

\[ W = f \cdot s \] where \( f \) is the force acting on an object (in Newtons), and \( s \) is the displacement of the object (metres); \( W \) is the work done (Joules).

Area of a parallelogram:

\[ |a \times b| \] is the area of the parallelogram formed by vectors \( a \) and \( b \); note that it is proven in your text book that \( |a \times b| = |a||b| \sin \theta \).

Torque:

- wrench turning; bicycle pedal; opening a door; anything with rotation around an axis caused by a force.
- depends on the radius (pushing on a door close to the hinges is a lot harder to open than by pushing on a door on the opposite side of the hinges.
- "righty tighty, lefty loosey" follows the right hand rule - the threads on screws are designed to follow the right hand rule
- units Nm [Newton metres], note that even though your text book uses Joules as a unit, that is not always viewed as an acceptable measure since Joules are a measure of Energy and torque is a force - it's one way to make that distinction.