MHF4U Review Practice Questions

1. Draw the reciprocal function on the same axes below:

- label graphs and axes
- max/min correspondance
- y intercept at 0.25
- points of intersection at ±1
2. Sketch the function \( f(x) = \frac{1}{(x+4)(x-3)(x+2)} \).

Sketch the function \( y = (x + 4)(x - 3)(x + 2) \) first to avoid having to check asymptote behaviour (you can tell from the graph if it's positive or negative) ** y-axis not to scale on the graph below:

- label graphs and axes
- max/min correspondence
- y intercept at \( \frac{1}{24} \)
- points of intersection at ±1

3. Complete the table:

<table>
<thead>
<tr>
<th></th>
<th>( f(x) )</th>
<th>( \frac{1}{f(x)} )</th>
<th>( x = -4 \text{ and } x = 6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x ) - intercepts</td>
<td>-4 and 6</td>
<td>VAs</td>
<td></td>
</tr>
<tr>
<td>( y ) - intercept</td>
<td>-24</td>
<td>y - intercept</td>
<td>( \frac{1}{24} )</td>
</tr>
<tr>
<td>positive intervals</td>
<td>(( -\infty, -4), (6, \infty))</td>
<td>positive intervals</td>
<td>(( -\infty, -4), (6, \infty))</td>
</tr>
<tr>
<td>negative intervals</td>
<td>((-4, 6))</td>
<td>negative intervals</td>
<td>((-4, 6))</td>
</tr>
<tr>
<td>increasing intervals</td>
<td>((1, \infty))</td>
<td>decreasing intervals</td>
<td>((1, \infty))</td>
</tr>
<tr>
<td>decreasing intervals</td>
<td>((-\infty, 1))</td>
<td>increasing intervals</td>
<td>((-\infty, 1))</td>
</tr>
<tr>
<td>local minimum</td>
<td>when ( x = 1 )</td>
<td>local maximum</td>
<td>when ( x = 1 )</td>
</tr>
</tbody>
</table>
Graph the function

\[ f(x) = \frac{12x - 36}{4x + 4} \]

- \( x - \text{int}: 3\)
- \( y - \text{int}: -9\)
- \( VA: x = -1\)
- \( HA: y = \frac{12}{4} = 3\)
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5. Graph the function
\[ f(x) = \frac{(x + 1)(x + 2)(x + 3)}{(x - 1)(x + 2)(x + 5)} \]

- x-intercept: \( x = -1, -3 \)
- y-intercept: \( y = -\frac{3}{5} \)
- hole: at \( x = -2 \)

Vertical Asymptotes (VA): \( x = 1, x = -5 \)

<table>
<thead>
<tr>
<th>Asymptote</th>
<th>Left</th>
<th>Right</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x = 1 )</td>
<td>( f(0.9) = -12.6 \ (down) )</td>
<td>( f(1.1) = 14.1 \ (up) )</td>
</tr>
<tr>
<td>( x = -5 )</td>
<td>( f(-5.1) = 14.1 \ (up) )</td>
<td>( f(-4.9) = -12.6 \ (down) )</td>
</tr>
</tbody>
</table>

Horizontal Asymptote (HA): \( y = \frac{1}{1} = 1 \)

- behaviour:
  - \( f(-100) = 1.0008 \ (above) \)
  - \( f(100) = 1.0008 \ (above) \)
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6. Graph the function
\[ f(x) = \frac{(x + 2)(x - 1)}{(x + 1)} \]

- **x - int:** -2, 1
- **y - int:** -2

**VA:** \( x = -1 \)

** behaviour:**

<table>
<thead>
<tr>
<th></th>
<th>Left</th>
<th>Right</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x = -1 )</td>
<td>( f(-1.1) = 18.9 \ (above) )</td>
<td>( f(-0.9) = -20.9 \ (below) )</td>
</tr>
</tbody>
</table>

**OA:**

\[ f(x) = \frac{x^2 + x - 2}{x + 1} \]

\[ -1 \]

\[ 1 \quad 1 \quad -2 \]

\[ -1 \quad 0 \quad -2R \]

\[ 1 \quad 0 \quad -2R \]

\[ f(x) = x - \frac{2}{x + 1} \]

Therefore, the **OA** is \( y = x \).

** behaviour:**

<table>
<thead>
<tr>
<th></th>
<th>Far left</th>
<th>Far right</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>( f(-100) = -99.98 )</td>
<td>( f(100) = 99.98 )</td>
</tr>
<tr>
<td>( y = x ) asymptote</td>
<td>-100</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>(-99.98 &gt; -100 \ (above))</td>
<td>(99.98 &lt; 100 \ (below))</td>
</tr>
</tbody>
</table>
7. Graph the function
\[ f(x) = \frac{(x + 3)(x + 1)}{(x + 1)(x + 4)(x - 2)} \]

- \( x - \text{int:} \quad -3 \)
- \( y - \text{int:} \quad -\frac{3}{8} \)

hole: at \( x = -1 \)

VA: \( x = -4, x = 2 \)

behaviour:

<table>
<thead>
<tr>
<th>Asymptote:</th>
<th>Left: ( f(-4.1) = -1.8 \ (\text{down}) )</th>
<th>Right: ( f(-3.9) = 1.5 \ (\text{up}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x = -4 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( x = 2 )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

HA: \( y = 0 \)

behaviour:

\( f(-100) = -0.0099 < 0 \ (\text{below}) \)
\( f(100) = 0.01 > 0 \ (\text{above}) \)
8. Mrs. Bethany bought some cupcakes for a bake sale for $18. When she wasn’t looking, Mr. Nathan ate one. She sold the rest at the bake sale for a total of $22, making a profit of $0.50 on each cupcake. How many cupcakes did Mrs. Bethany sell at the bake sale?

Let \( x \) be the number of cupcakes Mrs. Bethany sold at the bake sale, then \( x + 1 \) is the number of cupcakes originally purchased.

Unit buying price: \( \frac{18}{x + 1} \) ($/cupcake)

Unit selling price: \( \frac{22}{x} \) ($/cupcake)

Unit profit: \( \frac{1}{2} \) ($/cupcake)

\[
\begin{align*}
\frac{22}{x} - \frac{18}{x + 1} &= \frac{1}{2} \\
\frac{22(x + 1) - 18x}{x(x + 1)} &= \frac{1}{2} \\
\frac{22x + 22 - 18x}{x(x + 1)} &= \frac{2}{2} \\
\frac{4x + 22}{x(x + 1)} &= \frac{1}{2} \\
8x + 44 &= x(x + 1) \\
8x + 44 &= x^2 + x \\
0 &= x^2 - 7x - 44 \\
0 &= (x - 11)(x + 4) \\
x &= 11, -4 \text{ (inadmissible, can't have negative cupcakes)}
\end{align*}
\]

Therefore, Mrs. Bethany sold 11 cupcakes at the bake sale.
9. Mrs. Bethany can finish hanging a load of laundry 2 minutes faster than Mr. Nathan. Together, they can finish hanging a load of laundry in 7 minutes. How long does it take Mr. Nathan on his own?

*let* \( x \) *be the amount of time in minutes that Mr. Nathan takes on his own*

*then* \( x - 2 \) *is the amount of time in minutes that Mrs. Bethany takes on her own*

*Mrs. Bethany's rate: \( \frac{1}{x - 2} \) loads per minute*

*Mr. Nathan's rate: \( \frac{1}{x} \) loads per minute*

*Combined rate: \( \frac{1}{7} \) loads per minute*

\[
\frac{1}{x - 2} + \frac{1}{x} = \frac{1}{7}
\]

\[
\frac{x}{x(x - 2)} + \frac{x(x - 2)}{2x - 2} = \frac{1}{7}
\]

\[
\frac{x(x - 2)}{2x - 2} = \frac{1}{7}
\]

\[7(2x - 2) = x(x - 2)\]

\[14x - 14 = x^2 - 2x\]

\[0 = x^2 - 16x + 14\]

*quad formula:*

\[x = \frac{-(-16) \pm \sqrt{(-16)^2 - 4 \cdot 1 \cdot 14}}{2 \cdot 1}\]

\[x = \frac{16 \pm \sqrt{256 - 56}}{2}\]

\[x = \frac{16 \pm \sqrt{196}}{2}\]

\[x = \frac{16 \pm 14}{2}\]

\[x = 15.07, 0.93\]

*0.93 is inadmissible since it would result in a negative time for Mrs. Bethany*

*Therefore, it takes Mr. Nathan approximately 15 minutes to hang a load of laundry on his own.*

10. Describe what continuity is.

*A function is continuous if it does not contain any holes or VAs (or any break) over its whole domain.*

(can think of it as: you're able to draw the whole thing without lifting your pencil)
11. Solve the inequality:
\[
\frac{x^2 + 6x + 13}{x + 4} \leq \frac{20}{(x - 2)(x + 4)}
\]

\[
\frac{(x^2 + 6x + 13)(x - 2)}{(x + 4)(x - 2)} + \frac{20}{x^3 + 4x^2 + x - 26} \leq 0
\]

\[
\frac{20}{(x - 2)(x + 4)} + \frac{x^3 + 4x^2 + x - 6}{(x - 2)(x + 4)} \leq 0
\]

\[
factor \ n(x) = x^3 + 4x^2 + x - 6:
\]
\[
n(1) = 0, \text{ so } (x - 1) \text{ is a factor by the factor theorem}:
\]

\[
\begin{array}{cccccc}
1 & 4 & 1 & -6 \\
1 & 5 & 6 & 0R
\end{array}
\]

\[
n(x) = (x - 1)(x^2 + 5x + 6)
\]
\[
= (x - 1)(x + 2)(x + 3)
\]

continuing in the inequality:
\[
x^3 + 4x^2 + x - 6 \leq 0
\]
\[
\frac{(x - 2)(x + 4)}{(x - 1)(x + 2)(x + 3)} \leq 0; \ x \neq 2, -4
\]

find positive and negative intervals by graphing \( n(x) \) and \( d(x) = (x - 2)(x + 4) \)
(or by using a factor table)

\((-\infty, -4) \cup [-3, -2] \cup [1,2]\)

notice we use square brackets to show the inclusive inequality except on \( \infty \) and the restrictions.